## I Introduction

As technology rapidly grows up, AI industries are now the world’s hottest buzzword.

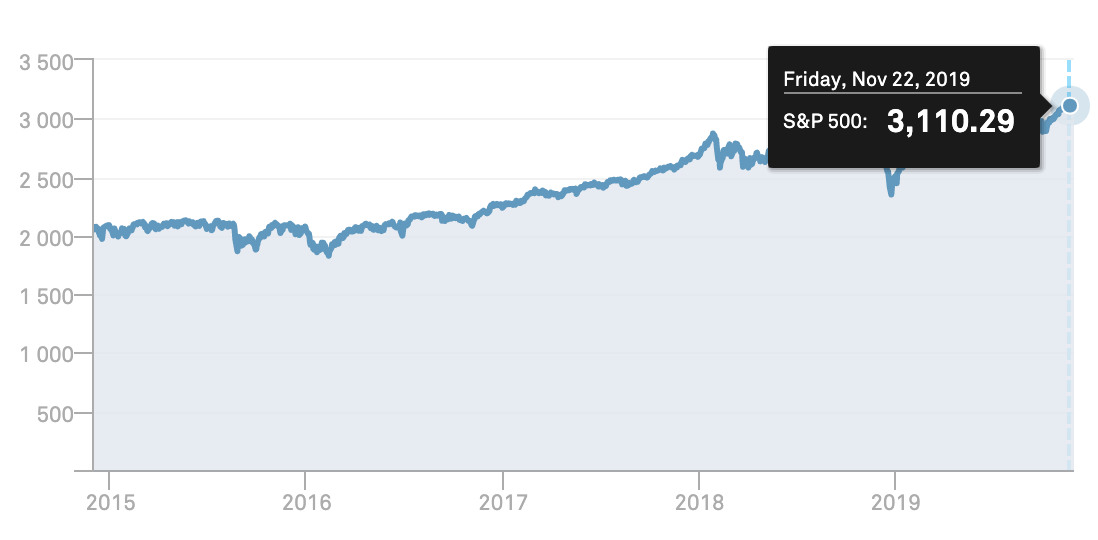
We selected five AI related stock to see whether it is as good as people said, and

compared them with Dow Jones U.S Technology index and S&P 500 to see the relationship

between AI-ETFs and the whole market.

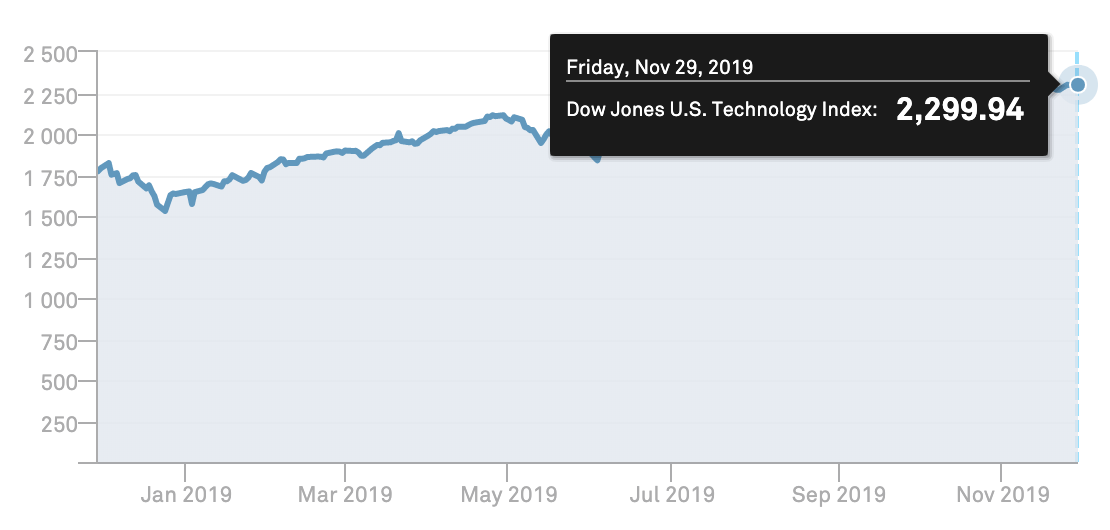
**S&P 500** (^GSPC)

The index includes 500 leading companies and covers approximately 80% of the US market shares, and the companies selected by S&P 500 must have larger or equal 50% revenue or fixed assets in the US. It is definitely an index that represents the whole stock market.



**Dow Jones U.S. Technology Index (**DJUSTC**)**

Dow Jones U.S Technology index is a member of Dow Jones Global Index, mainly focusing on measuring the U.S. technology companies’ performance.



These two market indexes fulfill the properties that they (1) large enough to be representatives of technology stocks and the whole stock market and (2) involve AI industries that can be compared with our target AI ETFs. By comparing S&P 500 and DJUST with our ETFs, we can have a glance at their relations and their performance relative to the whole industries and stock market.

1.

|  |  |  |
| --- | --- | --- |
| QQQ | | Tracking index: |
| **NASDAQ-100 Total Return** |
| **Market Cap** | **Return Last 3 Years** | **Return Last 12 months** |
| 73 billion | 75.87% | 101.34% |
| **Share by institution** | **Beta (36 month)** | **Dividend Yield Last year** |
| 41.7% | 1.13 | 0.76% |

QQQ has the largest market cap, and is the reason we choose it as a material to analyze.

2.

|  |  |  |
| --- | --- | --- |
| XLK | | Tracking index: |
| **The Technology Select Sector SPDR fund (XLK)** |
| **Market Cap** | **Return Last 3 Years** | **Return Last 12 months** |
| 5.07 billion | 29.68% | 13.22% |
| **Share by institution** | **Beta(36 month)** | **Dividend Yield Last year** |
| 85.36% | 1.02 | 2.02% |

XLK is again an AI related ETF but with its proportion investments on payments and technology companies (Visa, Master), the reason we choose XLK is that its shares highly held by institution.

3.

|  |  |  |
| --- | --- | --- |
| VGT | | Tracking index: |
| **MSCI US information Technology** |
| **Market Cap** | **Return Last 3 Years** | **Return Last 12 months** |
| 11.12 billion | 99.15% | 37.32% |
| **Share by institution** | **Beta (36 month)** | **Dividend Yield Last year** |
| 40.81% | 1.16 | 1.16% |

VGT with the highest return in last three years is the reason it is being picked.

4.

|  |  |  |
| --- | --- | --- |
| FDN | | Tracking index: |
| **Dow Jones Internet Composite Index** |
| **Market Cap** | **Return Last 3 Years** | **Return Last 12 months** |
| **1.79 billion** | **40.81%** | **6.5%** |
| **Share by institution** | **Beta(36 month)** | **Dividend Yield Last year** |
| **6.55%** | **1.46** | **0%** |

FDN with 0 dividend makes it so special to be picked up.

5.

|  |  |  |
| --- | --- | --- |
| IYW | | Tracking index: |
| **Dow Jones U.S. Technology Index** |
| **Market Cap** | **Return Last 3 Years** | **Return Last 12 months** |
| **9.55 billion** | **90.12%** | **36.46%** |
| **Share by institution** | **Beta(36 month)** | **Dividend Yield Last year** |
| **36.74%** | **1.15** | **0.74%** |

VGT is the only ETF that directly track our selected index, Dow Jones U.S. Technology Index.

The properties we mentioned above make us wondering how much will each property affects the ETF’s, how are their performances compare with our index, and the trending and distributions within them.

## II Data

### Collection of data

The data of the 5 ETF, S&P 500, and risk-free rate was collected form the Yahoo! Finance by function pdfetch\_YAHOO () in the R. The field we chose is the adjusted closing price (Adjusted close price adjusted for both dividends and splits) and the time interval is weekly. Data of the Dow jones U.S are obtained form the <https://us.spindices.com/indices/equity/dow-jones-us-technology-index>. Although we downloaded the daily data, we convert it to the weekly data in the R and make it exactly match the date of the data that we obtain from the YAHOO. In detail, the database covers the period January 1st, 2014 - October 1st,2019 and contains 300

observations weekly price. Then we adopt the diff () function in the R to get the log return based on these data. as the result of NA value, we finally got the 296 observations.

For the data or the Fama-French three factors model, we collected the data from the <https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html>. We also choice daily data first and then transfer to weekly data in order to matching the date.

### [Descriptive](javascript:;) [statistics](javascript:;)

After obtaining data, the first thing that we do is to better understand it. Thus, we adopt the R to conduct the [descriptive](javascript:;) [statistics](javascript:;). The whole the information we get from the R is displayed in the following table: According to the table, we separated them to 4 different segments based on the order of the moment.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| （％） | QQQ | XLK | VGT | FDN | IYW | SP500 | Dow J. tech |
| Mean | 0.2794 | 0.3082 | 0.3198 | 0.2714 | 0.3021 | 0.1627 | 0.2869 |
| median | 0.5679 | 0.5441 | 0.5116 | 0.4192 | 0.4686 | 0.3813 | 0.4850 |
| SD | 2.2804 | 2.2499 | 2.3560 | 2.7845 | 2.4152 | 1.7928 | 2.2637 |
| Var | 5.2007 | 5.0622 | 5.5508 | 7.7533 | 5.8332 | 3.2140 | 5.1242 |
| Min | -12.239 | -11.843 | -12.056 | -12.741 | -12.080 | -11.581 | -8.846 |
| Max | 7.5162 | 7.7273 | 7.4777 | 8.4474 | 7.4690 | 6.4144 | 5.9810 |
| range | 19.7544 | 19.5704 | 19.5340 | 21.1882 | 19.5494 | 17.9954 | 14.8265 |
| First Quantile | -0.7257 | -0.6181 | -0.7527 | -1.0487 | -0.7284 | -0.4866 | -0.7491 |
| Third Quantile | 1.5353 | 1.5786 | 1.7429 | 1.8037 | 1.7707 | 1.1413 | 1.6425 |
| Skew | -0.9803 | -0.9601 | -0.8803 | -0.8760 | -0.7785 | -1.4237 | -0.8326 |
| Kurt | 3.9284 | 4.0532 | 3.2137 | 3.1687 | 3.0007 | 7.7180 | 1.5188 |

#### First order moment

Based on the value of mean, we discovered means of three ETF, including VGT, XLK and IYW, are over than 0.3. But there is on significant difference among these 5 ETF. The FDN has smallest mean, 0.2714, and VGT has biggest means, 0.3198. At same times, we compared ETF’s mean with the S&P 500, all of them are over than it. But related to the Dow Jones U.S. tech index (hereinafter called DJUSTC), which is 0.2869, the mean of the FDN and that of the QQQ, which is 0.2794, are lower than it. For median, 0.5679 of QQQ occupy the first. The lowest one is SP500**.**

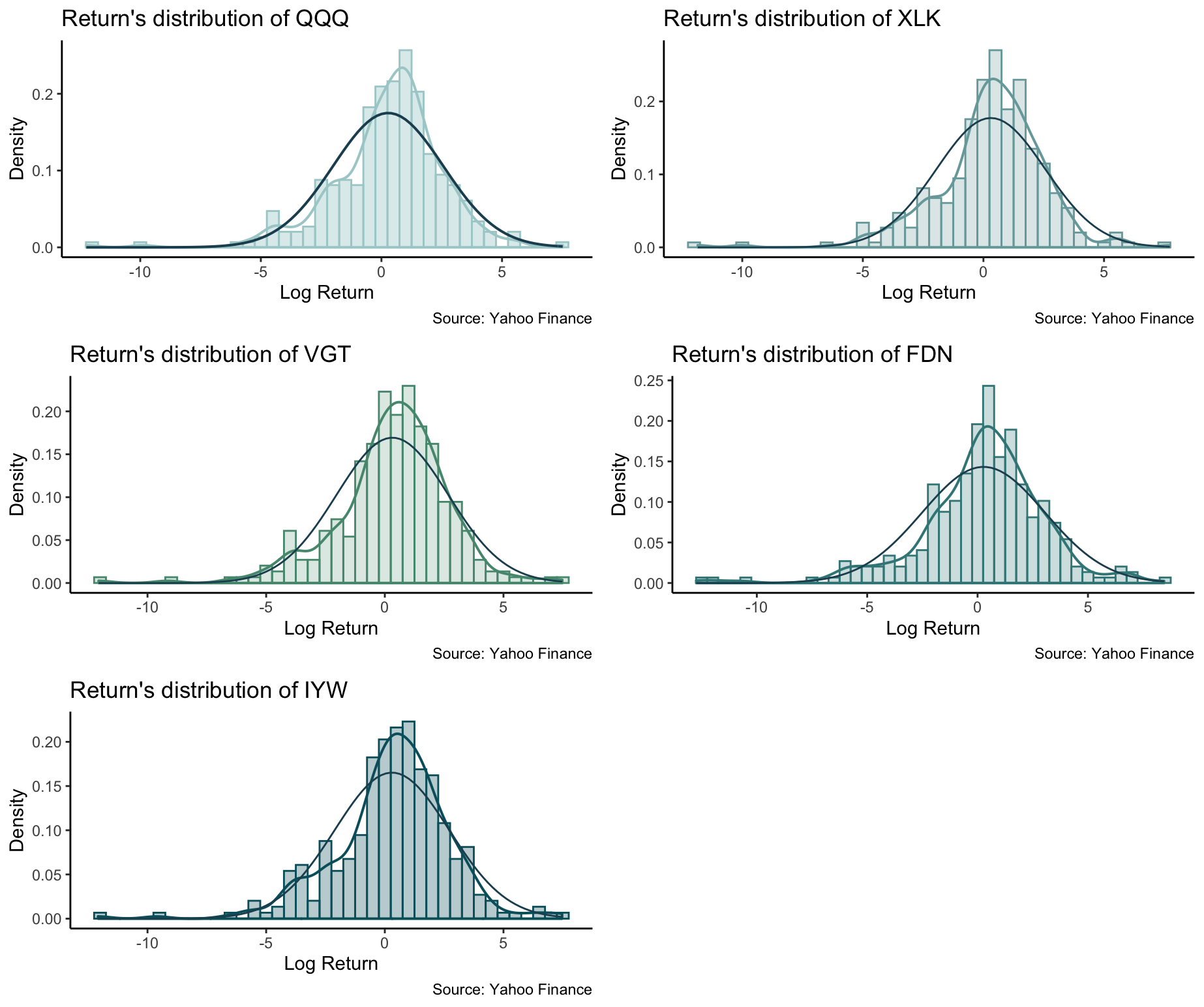
1. Second order moment

Second order moment can be used to describe the dispersion of the data. Except for XLK, the violations of the rest of ETFS are high than SP500 and DJUSTC. Specially, FDN has largest violation which is 7.7533. Furthermore, the violation can be used to measure the risk of company. Therefore, the investors of FDN may suffer the higher risk than the investors of other ETF.

1. Third and fourth moment

Negative skewness is common in equity index and stock return. Compared with normal distribution, these 5 ETFs are all moderate negative skewness, so they will have extreme value in the left-tail, and IYW has the skewness that is closest to zero in all the options, which means its mean is the one closest to the median from the left tail.

The ETFs’ kurtosis is larger than that of normal distribution, so as we learned before they will have a fatter tail, but DJUSTC has kurtosis smaller than 3, so DJUSTC will have fewer and less extreme outliner than normal distribution. S&P has less volatility and fewer mean return, but it is highly negative skewness and very leptokurtic. The statistics data reveal S&P is not widespread, and the return data are spread out from the center.



1. Variance/Covariance

After describing the data，we need to define the relationship between the index and ETF. The easiest way to do that is using variance and covariance. Covariance and correlation are two significantly used terms in the field of statistics and probability theory.

In previous section we mentioned a basic understanding of terms like means, standard deviation, correlations, sample sizes. Let us discuss a couple of terms more so that we can be more in depth.

Covariance measures the directional relationship between the returns on two [assets](https://www.investopedia.com/terms/a/asset.asp). A positive covariance means that asset returns move together while a negative covariance means they move inversely. In R, we can use Cov() function and we can get the below results.

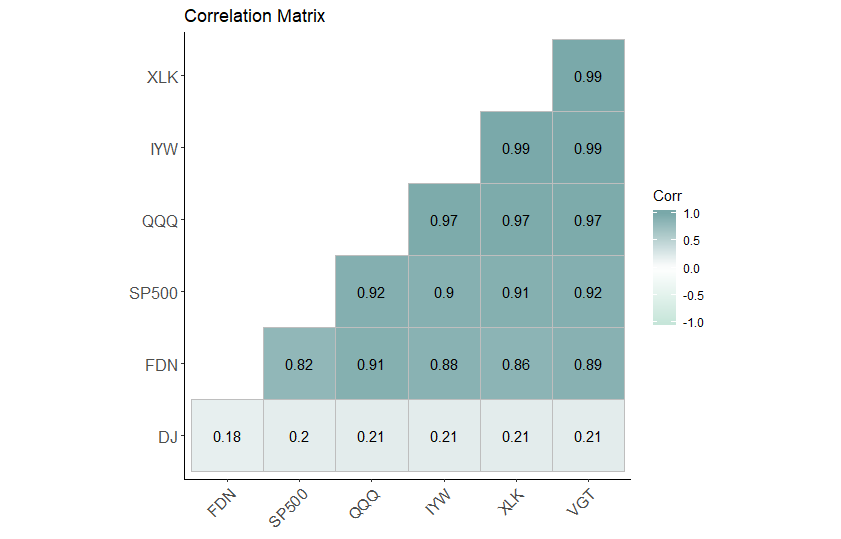
|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | QQQ | XLK | VGT | FND | IYW | SP500 | DJ |
| QQQ | 5.200669 | 4.977530 | 5.237738 | 5.776425 | 5.356675 | 3.777414 | 1.063537 |
| XLK | 4.977530 | 5.062236 | 5.236621 | 5.403471 | 5.359420 | 3.685798 | 1.050988 |
| VGT | 5.237738 | 5.236621 | 5.550802 | 5.844224 | 5.634764 | 3.869356 | 1.137255 |
| FND | 5.776425 | 5.403471 | 5.844224 | 7.753289 | 5.944192 | 4.097614 | 1.130976 |
| IYW | 5.356675 | 5.359420 | 5.634764 | 5.944192 | 5.833229 | 3.904098 | 1.133026 |
| SP500 | 3.777414 | 3.685798 | 3.869356 | 4.097614 | 3.904098 | 3.214081 | 0.829963 |
| DJ | 1.063537 | 1.050988 | 1.137255 | 1.130976 | 1.133026 | 0.829963 | 5.124152 |

From the picture, we can see all the covariances between the ETFs and the two indexes. However, it is difficult for us to observe the significance from covariance because the covariance is not standardized. We have to standardize covariance into correlation to observe it easier.

Correlation, in the finance and investment industries, is a statistic that measures the degree to which two securities move in relation to each other. It has a value that falls between -1.0 and +1.0. We use cor() function in R to easily calculate the correlations between the ETFs and the two indexes, the blow table is the results obtain from:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | QQQ | XLK | VGT | FND | IYW | SP500 | DJ |
| QQQ | 1.0000000 | 0.9700924 | 0.9748466 | 0.9096755 | 0.9725484 | 0.9239246 | 0.2060213 |
| XLK | 0.9700924 | 1.0000000 | 0.9878751 | 0.8624990 | 0.9862615 | 0.9137596 | 0.2063552 |
| VGT | 0.9748466 | 0.9878751 | 1.0000000 | 0.8908528 | 0.9902466 | 0.9160779 | 0.2132401 |
| FND | 0.9096755 | 0.8624990 | 0.8908528 | 1.0000000 | 0.8838842 | 0.8208421 | 0.1794318 |
| IYW | 0.9725484 | 0.9862615 | 0.9902466 | 0.8838842 | 1.0000000 | 0.9016495 | 0.2072403 |
| SP500 | 0.9239246 | 0.9137596 | 0.9160779 | 0.8208421 | 0.9016495 | 1.0000000 | 0.2045122 |
| DJ | 0.2060213 | 0.2063552 | 0.2132401 | 0.1794318 | 0.2072403 | 0.2045122 | 1.0000000 |

Except for the table, we also can use the graph to the results, in r we use the GGplot, the graph is displayed in the following:



Since there are no negative numbers in the output, generally, the ETFs and indexes move in the same way. As we can see, most of the ETFs have high correlation between each other (0.8 ~ 1.0). The reason is that all of the ETFs are related to technology. Same field of industry always leads to high correlation. It is also obvious that DJ’s correlation with others are relatively low (0.1 ~ 0.2). Although DJ index is also a technology index, the components of DJ index are not related to the ETFs we choose.

1. Tracking error

Then we mover forward to the tracking error, which is the [divergence](https://www.investopedia.com/terms/d/divergence.asp) between the price behavior of a position or a portfolio and the price behavior of a benchmark. This is often in the context of a [hedge fund](https://www.investopedia.com/terms/h/hedge.asp), [mutual fund](https://www.investopedia.com/terms/m/mutualfund.asp) or [exchange-traded fund](https://www.investopedia.com/terms/e/etf.asp) (ETF) that did not work as effectively as intended, creating instead an unexpected profit or loss. In the R, we can use TrackingError() function to obtain the tracking error, the result we shown below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Tracking Error** | | | | | |
|  | QQQ | XLK | VGT | FDN | IYM |
| TE\_SP500 | 6.687001 | 6.858971 | 7.304854 | 12.00631 | 8.027076 |
| TE\_DJ | 20.646617 | 20.503401 | 20.900315 | 23.49480 | 21.259095 |

Obviously, the tracking errors with DJ index in much higher than those with SP500 index. The conclusion is that, base on DJ, the ETFs are more likely to vary a lot in price.

1. Hypothesis test for the mean and variance

In order to know more about the difference between the five ETFs and the indexes, we will like to know if the mean and the variance of them are equal. Using T test and F test are good ways the achieve the goal.

An F-test (Snedecor and Cochran, 1983) is used to test if the variances of two populations are equal. The one-tailed version only tests in one direction, that is the variance from the first population is either greater than or less than (but not both) the second population variance. For our index and ETF, the null hypothesis of F-test is

: variances of ETF = variances of Index

In R, we use function var.test(), the p-value of the F -test is at below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | QQQ | XLK | VGT | FDN | IYM |
| S&P 500 | 0.00003962078 | 0.0001039048 | 0.000003166094 | 0.0000000000001016964 | 0.0000003831372 |
| DJUSTC | 0.89879725742 | 0.9169228346 | 0.492615165903 | 0.0003985101022445647 | 0.2662702166706 |

According to this table, we can conclude that the all the ETF has a different variance with the S&P500, and the DJUSTS only had the difference variances with FDN

Based on this conclusion we obtain from the F-test, we can identify which type of t-test to use for measure means of index whether equal to means of EFT.

In t test, there are two different type, one is the two variances are different and the other is that the variances are same. We have to do F test first to determine whether their variances are equal or not, and then do the T test. We can set that H0 is that the ETF’s mean is equal to the index’s mean. Ha is that the means are not equal. And The default confidence is 95%, so that alpha = 0.05. In R, we use T.test() function, and the result is in the following:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | QQQ | XLK | VGT | FDN | IYM |
| S&P 500 | 0.4889195 | 0.3843942 | 0.3615518 | 0.5725066 | 0.4254676 |
| DJUSTC | 0.9680972 | 0.9084967 | 0.8624909 | 0.9406892 | 0.9370255 |

According to this table, the all p-value of t-test is large 0.05. Therefore, the mean of ETF is equal to the index’s mean.

III. Model

After analyzing the data, we start to building the model. The first model we used is based on the (capital asset pricing model) CAMP, which is developed by [Sharpe (1964)](https://onlinelibrary.wiley.com/doi/full/10.1111/j.1755-053X.2011.01147.x?casa_token=6UBHwTUVVLAAAAAA%3AaoYiE5293J-qEN08X-2eOk3xbo5oTmvnAkOT2lgNLMFf_EXfEtt3ubVjdxPDfOgUZk933QaG1n4vqkk#b47), [Lintner (1965)](https://onlinelibrary.wiley.com/doi/full/10.1111/j.1755-053X.2011.01147.x?casa_token=6UBHwTUVVLAAAAAA%3AaoYiE5293J-qEN08X-2eOk3xbo5oTmvnAkOT2lgNLMFf_EXfEtt3ubVjdxPDfOgUZk933QaG1n4vqkk#b36), and [Mossin (1966)](https://onlinelibrary.wiley.com/doi/full/10.1111/j.1755-053X.2011.01147.x?casa_token=6UBHwTUVVLAAAAAA%3AaoYiE5293J-qEN08X-2eOk3xbo5oTmvnAkOT2lgNLMFf_EXfEtt3ubVjdxPDfOgUZk933QaG1n4vqkk#b39). This model can help us to determine the required rate of return of an asset depend on the systematic risk measure, betas. In the model, betas is the [sensitivity](https://en.wikipedia.org/wiki/Sensitivity_and_specificity) of the expected excess asset returns to the expected excess market returns. According to this idea, we use the univariate regression model to determine the beta and to compute the linear relationship between our funds and index. we use weekly returns of S&P 500 index and Dow Jones Technology index as the independent variables to define the linear relationship between the excess returns of each ETF and the two indexes, respectively. The model that we use is in the following:

In detail, is the excess return of each fund, and is the excess return of S&P 500 index or Dow Jones Technology Index.

Another model we used is Fama-French 3 factors model, which was proposed by [Fama](https://www.sciencedirect.com/science/article/abs/pii/0304405X93900235" \l "!) and [French](https://www.sciencedirect.com/science/article/abs/pii/0304405X93900235" \l "!) in 1993. [Fama](https://www.sciencedirect.com/science/article/abs/pii/0304405X93900235#!) and [French](https://www.sciencedirect.com/science/article/abs/pii/0304405X93900235#!)(1993) discovered that the companies with high book‐to‐market are more likely to be in financial distress, and the small capitalization stocks may be more sensitive to changes in business, so these firms actually may have higher returns than those predicted by the CAPM. Therefore, excepting the market risk, this model incudes other 2 factors, a size factor (SMB) and a value factor (HML):

The SMB is the equal-weight average of the returns on the three small stock portfolios for the region minus the average of the returns on the three big stock portfolios. HML is the equal-weight average of the returns for the two high B/M portfolios for a region minus the average of the returns for the two low B/M portfolios.

**III Assumptions**

A1. For the univariate regression model, the basic assumption we made is that our indexes, including S&P 500 index and Dow Jones U.S. Technology Index, have linear relationship with our 5 different ETF - QQQ, XLK, VGT, FDN, and IYW. For the Fama-French three factor model, we also assume that our five ETF has linear relation with the market risk, the outperformance of small versus big companies, and the outperformance of high [market](https://en.wikipedia.org/wiki/Book-to-market_ratio) versus small market companies.

A2. For any realization of X, we assumed that the expected value of is zero. We imply that in our model, our two indexes in the univariate regression model and three factors in the Fama-French three factor model are independent with the error term.

A3. we assumed that all of the regressors in the model are non-stochastic.

A4．We assumed that the variance of the error should be the normal distribution

A5. The other two assumptions related to the Gauss-Markov theorem, including spherical disturbance assumption and no strict multicollinearity, are not automatically assumed. In order to establish an acceptable model, different tests were conducted to examine each model’s heteroscedasticity, autocorrelation and multicollinearity.

**A)** ***Heteroscedasticity*.**

***1. Detecting Heteroscedasticity*.**

***1.1) Breusch–Pagan test***

There are two different tests that we can use. First one is Breusch–Pagan test that developed by Breusch and Pagan in 1979, and null hypothesis is model with homoskedasticity. In addition, it compares Lagrange Multiplier (LM) and chi-square to make decision where reject null hypothesis. In 1981, Koenker discovered that Breusch–Pagan test is vulnerable to violation of the normality assumption, so he adopted i.i.d. assumption. Thereafter, Koenke (1981) adopted the” Studentized LM”, which is a modification of LM.

***1.2) White test***

Second test is White test developed by White in 1980. As the result of missing the nonlinear relationship in the BP test, the White test overcame this weakness by containing squares and cross-products of independent variables in the [auxiliary](javascript:;) [regression](javascript:;).

***2*. Dealing with** **heteroskedasticity**

**2.1 Robust Standard Errors**.

As we mentioned before, heteroskedasticity causes standard errors to be biased. OLS assumes error of model are both independent and identically distributed. In this situation, we can use robust standard errors that relax this assumption. Therefore, we can still use OLS but adopt the robust standard errors to make our model more trustworthy.

**2.2 Feasibility Weighted Least Squares (FWLS).**

FWLS is more advanced but more difficult to use. The reason is that the FWLS need to base on the auxiliary regression to estimate variance of error, but it is difficult to precisely identify the function of auxiliary regression. However, if we can make it work right, it will be superior than using robust standard errors.

***3. Results*** **heteroskedasticity**

**3.1 Test choice- White test**

For our study, we adopted the White test to detect the heteroscedasticity, because after add quadratic and cross-product interactions, we can test higher order of heteroskedasticity for our model. Furthermore, in our model, the number of variables is small, so we can overcome the White test limitation that quadratic and interaction terms will become unwieldy when many regressors are present in the model.

**3.2 R - White test**

In R, we use test function bptest() from package lmtest to carry out White’s test. Even though the title of results in the R is studentized Breusch-Pagan test, but we can add the quadratic term to the function bptest () to cover them to be White test. For example, for the linear regression of QQQ, our White’s test function is bptest (lm\_QQQ\_SP500, ~ I(SP500^2), data = returns\_ex). The I(SP500^2) is the quadratic term that we add. However, for the Fame-French 3 factor model, we have to add the cross-product, such as SMB\*HML.

**3.3 Result- White test**

The result of White’s test in the following table:

|  |  |  |
| --- | --- | --- |
| ***S&P 500*** | ***Dow Jones U.S. Technology Index*** | ***Fama-French 3 factors*** |
| *studentized Breusch-Pagan test*  *data: lm\_QQQ\_SP500*  *BP = 2.6375, df = 1, p-value = 0.1044* | *studentized Breusch-Pagan test*  *data: lm\_QQQ\_DJ*  *BP = 4.0805, df = 1, p-value = 0.04338* | *studentized Breusch-Pagan test*  *data: lm\_QQQ\_ff*  *BP = 5.9239, df = 9, p-value = 0.7475* |
| *studentized Breusch-Pagan test*  *data: lm\_XLK\_SP500*  *BP = 0.827, df = 1, p-value = 0.3631* | *studentized Breusch-Pagan test*  *data: lm\_XLK\_DJ*  *BP = 2.7595, df = 1, p-value = 0.09668* | *studentized Breusch-Pagan test*  *data: lm\_XLK\_ff*  *BP = 5.5269, df = 9, p-value = 0.7862* |
| *studentized Breusch-Pagan test*  *data: lm\_VGT\_SP500*  *BP = 1.6741, df = 1, p-value = 0.1957* | *studentized Breusch-Pagan test*  *data: lm\_VGT\_DJ*  *BP = 2.3638, df = 1, p-value = 0.1242* | *studentized Breusch-Pagan test*  *data: lm\_VGT\_ff*  *BP = 5.3415, df = 9, p-value = 0.8036* |
| *studentized Breusch-Pagan test*  *data: lm\_FDN\_SP500*  *BP = 1.4292, df = 1, p-value = 0.2319* | *studentized Breusch-Pagan test*  *data: lm\_FDN\_DJ*  *BP = 3.0846, df = 1, p-value = 0.07904* | *studentized Breusch-Pagan test*  *data: lm\_FDN\_ff*  *BP = 3.7571, df = 9, p-value = 0.9267* |
| *studentized Breusch-Pagan test*  *data: lm\_IYW\_SP500*  *BP = 2.5058, df = 1, p-value = 0.1134* | *studentized Breusch-Pagan test*  *data: lm\_IYW\_DJ*  *BP = 2.4541, df = 1, p-value = 0.1172* | *studentized Breusch-Pagan test*  *data: lm\_IYW\_ff*  *BP = 5.0836, df = 9, p-value = 0.827* |

The null hypothesis of the White test is existing heteroscedasticity. In the above table, we can see that no one p-value is less than 0.05, thus we cannot reject their null hypotheses. In this situation, all of our model is heteroscedasticity homoscedasticity.

**B) *Autocorrelation***

***1. Detecting Autocorrelation***

***1.1******Breusch–Godfrey test.***

Breusch–Godfrey test is proposed by Breusch and Godfrey ([1978](https://www.tandfonline.com/doi/full/10.1080/13504850701748933?casa_token=dzEMMmyx2j8AAAAA%3A-adaQryexqvTxdAlTq5Ycpu2D6KFzJyfvFfk7tomZv65pXz7WmZ0B3kvz--q6ua5cGp0HPwcy1M3)). As the result of Lagrange multiplier testing that it based on, it also called LM test. It uses the sample residuals to run in the auxiliary regression model and then to test the null hypothesis. The null hypothesis is no [serial correlation](https://en.wikipedia.org/wiki/Serial_correlation) of any order up to p. Davidson and MacKinnon (1993) based on the Breusch–Godfrey test toproposed other approach***.*** They replaced the missing values in the initial observations on the lagged residuals in the auxiliary regression with zeros. Therefore, the different between these two approaches is degrees of freedom of a Lagrange-multiplier model.

**1.2 Q-Test**

*Box and Pierce,1970* developed the Q-Test. The key different with the Breusch–Godfrey test is that Q-Test based on the correlation of the error to test autocorrelation. It also can be used to test higher order of autocorrelation. The null hypothesis is the absence of serial autocorrelation. In 1979 the Ljung and Box modified this test to make it more suitable the small sample size.

**1.3 Durbin-Watson test**

The DW test is proposed by Durbin and Watson in 1950. Unlike the Breusch–Godfrey test and Q-Test, it can only used to test first-order autocorrelation, which is one of its weakness. In addition. Its range is from 0 to 4. When DW value equal to 2, there is no autocorrelation. But DW test sometime may be inconclusive.

**2**. **Dealing with** **autocorrelation**

**2.1 Heteroskedasticity and** **autocorrelation consistent standard error (HAC)**

Heteroskedasticity and autocorrelation consistent standard error, which also call Newey-West standard error, is developed by the Newey and West in 1987. Like heteroskedasticity, autocorrelation just influence our standard error of the model. It will made it unbiased. In this situation, we can use HAC in the OLS, which can help us to solve the both Heteroskedasticity and autocorrelation.

**2.2** **Quasi-experiments approach**

In 1940, [Donald Cochrane](https://en.wikipedia.org/wiki/Donald_Cochrane_(economist)) and [Guy Orcutt](https://en.wikipedia.org/wiki/Guy_Orcutt) developed *the* Cochrane–Orcutt procedure. The key idea is to transform the model to eliminate the autocorrelation by taking a quasi-difference method. Through this way, the error term will translate to the white noise, which do not have the autocorrelation. THEN in 1954, the [Sigbert Prais](https://en.wikipedia.org/wiki/Sigbert_Prais" \o "Sigbert Prais) and [Christopher Winsten](https://en.wikipedia.org/w/index.php?title=Christopher_Winsten&action=edit&redlink=1) modified this model, because it lose the first observation. Prais–Winsten estimation add this observation back to make this approach more efficiency.

**2.3** **Feasible general least square (F****GLS)**

In fact, the Prais–Winsten estimation is a special case of the FGLS. When our model has autocorrelation and heteroskedasticity, we can depend on the GLS to change our original model to obtain a new model in which there is no autocorrelation in the error terms. Then we can base on the OLS to estimate the parameters for the new model. But sometimes it is difficulty to define the function of parameters. If we cannot precisely estimate it, our model will be the problem.

**3. Results and analysis – autocorrelation**

**3.1 Test choice- Breusch–Godfrey test**

Compare with the Durbin-Watson test, Breusch–Godfrey test and Q test will be more suitable for our model. As we mentioned above, there are several weaknesses of Durbin-Watson test. Sometime we even cannot get the conclusion. For the Breusch–Godfrey test and Q test, they have the similar nature. The different is the variables in the null hypothesis, but both null hypothesizes are no [autocorrelation](https://en.wikipedia.org/wiki/Serial_correlation) of any order up to p. For our model, we decide to choose Breusch–Godfrey test, which more convivence to use. Because in R, the argument in the function Box.test (), which used to conduct Q-test, should be a numeric vector or univariate time series. However the bgtest (), which for the Breusch–Godfrey test, can directly use a lm() as the argument.

**3.2 R - Breusch–Godfrey test**

The function bgtest（）in R is provided by the “lm” package. Excepting the formula that we need to test, there are parameters in the several arguments can be choose. For example, we can use the maximal order of serial correlation to be tested by setting the “order” argument. And also we can use “type” to define type of test statistic. However, in our model, we just keep all the parameters as the default of the bgtset ().

**3.3 Result - Breusch–Godfrey test**

|  |  |  |
| --- | --- | --- |
| ***S&P 500*** | ***Dow Jones U.S. Technology Index*** | ***Fama-French 3 factors*** |
| *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_QQQ\_SP500*  *LM test = 1.2989, df = 1, p-value = 0.2544* | *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_QQQ\_DJ*  *LM test = 19.022, df = 1, p-value = 0.00001292* | *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_QQQ\_ff*  *LM test = 0.1195, df = 1, p-value = 0.7296* |
| *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_XLK\_SP500*  *LM test = 2.0188, df = 1, p-value = 0.1554* | *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_XLK\_DJ*  *LM test = 22.737, df = 1, p-value = 0.000001858* | *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_XLK\_ff*  *LM test = 0.13226, df = 1, p-value = 0.7161* |
| *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_VGT\_SP500*  *LM test = 0.079286, df = 1, p-value = 0.7783* | *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_VGT\_DJ*  *LM test = 21.231, df = 1, p-value = 0.000004072* | *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_VGT\_ff*  *LM test = 0.26707, df = 1, p-value = 0.6053* |
| *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_FDN\_SP500*  *LM test = 0.048425, df = 1, p-value = 0.8258* | *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_FDN\_DJ*  *LM test = 12.456, df = 1, p-value = 0.0004166* | *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_FDN\_ff*  *LM test = 0.27587, df = 1, p-value = 0.5994* |
| *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_IYW\_SP500*  *LM test = 0.083466, df = 1, p-value = 0.7727* | *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_IYW\_DJ*  *LM test = 22.742, df = 1, p-value = 0.000001853* | *Breusch-Godfrey test for serial correlation of order up to 1*  *data: lm\_IYW\_ff*  *LM test = 0.081869, df = 1, p-value = 0.7748* |

The null hypothesis of Breusch–Godfrey test is there is no autocorrelation, we can make conclusion by compare with p-value and the 0.05. We can see that all of the single regression model related to S&P 500 index and to Fama-French 3 factors do not have autocorrelation. On the other head, based on the DJ U.S Tech index, all of the model exists the autocorrelation.

**3.3 Remedy – HAC**

In our model, when we detect the autocorrelation, we adopt still use the OLS with the HAC. Autocorrelation, like autocorrelation, just influence our standard error. If we use HAC, which is unbiased, to replace our original error, our model will fix the autocorrelation problem. although the FGLS is a more superior method we did not choose the quasi-differing method and FGLS. Because it is difficult for us to identify the function of auxiliary regression:

In the R, we adopt coeftest() function from the “lmtest” package. In the function, we need based on the ‘vcov’, which is an argument in the function. We assign the HAC to the vcov, and then we can applied it to our model. For getting HAC, we need other function, NeweyWest(). Depending on this function, we can get the HAC covariance matrix estimators. For example, coeftest(lm\_QQQ\_DJ, vcov = NeweyWest(lm\_QQQ\_DJ)) is the function we used to solve the autocorrelation in the regression model of QQQ related to the DJ Tech index. And the result of the Remedy, we will show in the next part- analysis.

**C) Multicollinearity**

**1. Detecting multicollinearity**

Variance inflation factor (VIF) is based on the , which is the [multiple R2](https://en.wikipedia.org/wiki/Coefficient_of_determination) for the regression of on the other covariates. The 1 / (1 − *Rj*2) is the VIF, which range of value from 1 upward. There is no strict standard to make the conclusion, but the empirical rule is over 10. It means that if our VIF over than 10, we can conclude that there is Multicollinearity.

**2****. Dealing with** **multicollinearity**

**2.1 Do nothing**

Sometimes, even though our model exist multicollinearity. But we can ignore this situation. This action should depend on our goal. If we do not care about THE coefficient of each independent variable, we just want to use our model to predict the dependent variable. In this situation, even though the multicollinearity would influence the coefficient, the whole model still can be use to predict. Another example is the variables with high VIFs are control variables, and the variables of interest do not have high VIFs. Therefore, we do need to fix all the multicollinearity.

**2.2 Remove**

If our goal is to measure the accurate coefficient, we have to solve the multicollinearity. Unlike the autocorrelation and heteroskedasticity, which just affect the standard error, the multicollinearity has effect on the coefficients. Therefore, we can remove some of the highly correlated independent variables. In addition, we also can combine the high correlated the independent variables.

**3. Results and analysis – multicollinearity**

In the R, we use vif() function form “[car](https://www.rdocumentation.org/packages/car/versions/3.0-5)” package. In our model, we just need to test our Fama-French 3 factors model. Take the QQQ as the example, in R we use fuction vif(lm\_QQQ\_ff), and the lm\_QQQ\_ff is a variable that represents the regression model. The result of vif() show below. Since our Fama-French three factors have same independent variable, we just need to use VIF method to test one ETF. The result of VIF is show below.

|  |  |  |  |
| --- | --- | --- | --- |
| **VIF** | Mkt.RF | SMB | HML |
| Fama-French 3 factors | 1.067519 | 1.056056 | 1.036814 |

All VIF is very small. According to our empirical rule, there is no multicollinearity in our model.

iv. Analysis

In the analysis part, we will discuss the results that we obtain from the linear regression model by using R language. we mainly force on the t-test and R square. R square is the useful statistical measure to provide the information of model’s goodness of fit. In the regression model, we can use the explained sum of squares (ESS) divided by the total sum of square (TSS) to compute the . The function is in the following:

According to this formula, we can see the can tell us how many presents of the data the model can explain. Thus, we will use R square to measure our model first,

Then, t-test used to test the significant of the independent variable. The null hypothesis is：

If we reject the null hypothesis, the independent variable is significant. t-test play the important role in our model. Without it, even though we obtain the coefficient, we cannot make sure this independent variable can be properly to predict the dependent variable. Therefore, what’s why we primary based on t-test and R square to analyze our model.

1 S&P 500

When we use S&P 500 as a regression parameter, our model can obtain an ideal result. We can see that each regression model of the five ETF has the high R square. In detail, QQQ has the highest R square, which is 0.8537. The he regression model of the QQQ explains the data more completely. On other head, the lowest one is FDN, 0.6. But in the finance world, such R square is not low. Therefore, as the perspective of the whole model, the regression model is enough used to explain the data. For t-test, we will discuss the model one by one.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **QQQ-S&P 500** | | | | | |
| R-squared | 0.8537 |  |  |  |  |
| Adj R-squared | 0.8532 |  |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** | |
|  | 0.09110 | 0.05096 | 1.788 | 0.0749 | |
|  | 1.17535 | 0.02838 | 41.417 | <2e-16 \*\*\* | |
|  | | | | | |

Compare with other ETF, the QQQ has the highest R square, which is 0.8537. as the perspective of the whole model, the regression model is adequate used to explain the data. For the QQQ, we can see that p value of is 0.0749, which is higher than 0.05. Thus, the is not a significant parameter. However, the is smaller than , so the is a significant parameter.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **IYW – S&P 500** | | | | |
| R-squared | 0.813 |  |  |  |
| Adj R-squared | 0.8123 |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** |
|  | 0.10804 | 0.06102 | 1.771 | 0.0777 |
|  | 1.21464 | 0.03398 | 35.748 | <2e-16 \*\*\* |
|  | | | | |

Like the QQQ, the α is not significant but β is significant. In this situation, we cannot directly remove the α. Because if we remove it, when excess return of index to be zero, our ETF will equal to the risk-free rate. In the reality, this situation is rare. Sometimes, removing it will let our model unreasonable

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **VGT - S&P 500** | | | | |
| R-squared | 0.8391 |  |  |
| Adj R-squared | 0.8386 |  |  |  | |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** | |
|  | 0.12733 | 0.05520 | 2.307 | 0.0218 \* | |
|  | 1.20371 | 0.03074 | 39.162 | <2e-16 \*\*\* | |
|  | | | | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **XLK -S&P 500** | | | | |
| R-squared | 0.8349 |  |  |  |
| Adj R-squared | 0.8344 |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** |
|  | 0.12410 | 0.05340 | 2.324 | 0.0208 \* |
|  | 1.14669 | 0.02973 | 38.564 | <2e-16 \*\*\* |
|  | | | | |

Unlike QQQ, both XLK and VGT’s are significant for the model, but such significant live not we strong. We can define the level of the significant by counting “\*”. More “\*” means p value more small

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **FDN - S&P 500** | | | | | |
| R-squared | 0.6738 |  |  |  |  |
| Adj R-squared | 0.6727 |  |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** | |
|  | 0.068483 | 0.096050 | 0.713 | 0.4764 | |
|  | 1.274919 | 0.065940 | 19.335 | <2e-16 \*\*\* | |
|  | | | | | |

For the FDN, although its R-square is lowest, but is still significant. There is no obviously relationship between the R-square and t-statistics. Thus, sometime we may discover that our R-squares are very small but the variables have strong significant level. This situation, we will obvious in the DJ Tech index part

**2. DJ Tech index**

The below tables have 2 different values in the column of standard error, t-test, and p-value. The reason for that is autocorrelation. Our regression models based on the DJ Tech index are conflict with spherical disturbance. In this situation, we adopted the HAC to remedy our model. For this reason, our standard errors are changed. The value in the parentheses () is the value before the remedy. We can see that after dealing with the autocorrelation, the t-statistics of the independent variables decrease, which reduce our significant level of variables. But our model become more acceptable.

Moreover, the none of is over than 0.05. The goodness of fit is weak. Which mean the model just can explain less than 5% of data. But the interesting thing is that our coefficient of still significant. Actually, in the finance world, we generally cannot measure many factors. In this situation, the error term will be dominate, which causes the low . Therefore, when this situation happens, the t-test are more useful.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **QQQ-DJ** | | | | |
| R-squared | 0.04247 |  |  |  |
| Adj R-squared | 0.03921 |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** |
|  | 0.206890 | 0.112474  (0.1309) | 1.8395  (1.581) | 0.066857  (0.114968) |
|  | 0.207637 | 0.067567  (0.0575) | 3.0731  (3.611) | 0.002317 \*\*  (0.000358 \*\*\*) |
|  | | | | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **XLK - DJ** | | | | |
| R-squared | 0.04257 |  |  |
| Adj R-squared | 0.03932 |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** |
|  | 0.236384 | 0.099542  (0.12909) | 2.3747  (1.831) | 0.018204 \*  (0.068094) |
|  | 0.205085 | 0.061839  (0.05672) | 3.3164  (3.616) | 0.001026 \*\*  (0.000352 \*\*\*) |
|  | | | | |
|  | | | | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **FDN - DJ** | | | | |
| R-squared | 0.03221 |  |  |
| Adj R-squared | 0.02892 |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** |
|  | 0.195285 | 0.127576  (0.16063) | 1.5307  (1.216) | 0.126909  (0.22506) |
|  | 0.220767 | 0.081613  (0.07058) | 2.7051  (3.128) | 0.007228 \*\*  (0.00194 \*\*) |
|  | | | | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **IYW -DJ** | | | | |
| R-squared | 0.04295 |  |  |  |
| Adj R-squared | 0.03969 |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** |
|  | 0.225924 | 0.119214  (0.13855) | 1.8951  (1.631) | 0.059057  (0.104039) |
|  | 0.221117 | 0.066257  (0.06088) | 3.3372  (3.632) | 0.000955 \*\*\*  (0.000331 \*\*\*) |
|  | | | | |

According to above 3 tables, the significant level of the coefficients of QQQ, XLK, and FDN has decrease. But they still keep the strong significant. On the other hand, all p-value of t-test for are smaller than 0.05. The coefficients of are not significant.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **VGT - DJ** | | | | |
| R-squared | 0.04545 |  |  |  |
| Adj R-squared | 0.0422 |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** |
|  | 0.243420 | 0.117131  (0.1350) | 2.0782  (1.804) | 0.0385601 \*  (0.07233 ) |
|  | 0.221867 | 0.065502  (0.0593) | 3.3872  (3.741) | 0.0008024 \*\*\*  (0.00022 \*\*\*) |
|  | | | | | |

For the regression model of VGT, it is only one the and both are significant. And even the is more significant than other four ETF. At same time, it has highest , which means the DJ U.S. index can provide more information about the VGT than other ETF.

**3. Fama-French three factors**

Excepting testing the Heteroskedasticity and autocorrelation, Fama-French as the multiple linear regression also need to check the Multicollinearity. But here, we do not have the problem of Multicollinearity. And following is the data we collect from the regression mode.

Other situations, we need pay attention. In the multiply linear regression model, we use adjusted to replace the . The reason is that, if we still use , when we add the any variable into our model, will increase. Even this variable does not have relationship with our dependent variable. We can see the function of adjusted is:

adjusted

therefore, the adjusted consider the degree of freedom, so it can solve the problem when we add the unrelated variable in to the model.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **QQQ-Fama** | | | | |
| R-squared | 0.1751 |  |  |  |
| Adj R-squared | 0.1667 |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** |
|  | 0.16039 | 0.12205 | 1.314 | 0.18982 |
|  | 0.97663 | 0.14444 | 6.761 | 7.44e-11 \*\*\* |
| SMB | -0.02671 | 0.23740 | -0.113 | 0.91049 |
| HML | -0.61557 | 0.23005 | -2.676 | -0.00788 \*\* |
|  | | | | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **XLK-Fama** | | | | |
| R-squared | 0.1597 |  |  |  |
| Adj R-squared | 0.151 |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** |
|  | 0.1941 | 0.1215 | 1.597 | 0.1114 |
|  | 0.9540 | 0.1438 | 6.632 | 1.6e-10 \*\*\* |
| SMB | -0.1090 | 0.2364 | -0.461 | 0.6450 |
| HML | -0.5010 | 0.2291 | -2.187 | 0.0296 \* |
|  | | | | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **VGT – Fama** | | | | |
| R-squared | 0.1618 |  |  |  |
| Adj R-squared | 0.1532 |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** |
|  | 0.20123 | 0.12710 | 1.583 | 0.1145 |
|  | 0.98645 | 0.15043 | 6.558 | 2.48e-10 \*\*\* |
| SMB | -0.02886 | 0.24723 | -0.117 | 0.9072 |
| HML | -0.55857 | 0.23958 | -2.331 | 0.0204 \* |
|  | | | | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **FDN – Fama** | | | | |
| R-squared | 0.1552 |  |  |  |
| Adj R-squared | 0.1465 |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** |
|  | 0.1437 | 0.1508 | 0.953 | 0.3413 |
|  | 1.0899 | 0.1785 | 6.106 | 3.25e-09 \*\*\* |
| SMB | 0.3412 | 0.2933 | 1.163 | 0.2457 |
| HML | -0.5815 | 0.2843 - | 2.046 | 0.0417 \* |
|  | | | | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **LYW – Fama** | | | | |
| R-squared | 0.1597 |  |  |  |
| Adj R-squared | 0.1511 |  |  |  |
| Regression Output | **Coef.** | **Std. Error** | **t-test** | **P** |
|  | 0.18212 | 0.13046 | 1.396 | 0.164 |
|  | 1.02059 | 0.15440 | 6.610 | 1.82e-10 \*\*\* |
| SMB | 0.02027 | 0.25376 | 0.080 | 0.936 |
| HML | -0.48820 | 0.24591 | -1.985 | 0.048 \* |
|  | | | | |

According to the above table, we discovered that, the adjusted R square is low when we compare with model based on the S&P500. In addition, the Fama-French three factor model have three variables. All of ETF, their excess turn on the market and [HML](https://www.investopedia.com/terms/h/high_minus_low.asp) (high minus low) are significant for the model. But [SMB](https://www.investopedia.com/terms/s/small_minus_big.asp) (small minus big) is not significant. In this case, even though we drop this variable out from our model. There is no significant influence on the model.

V Conclusion

|  |  |  |
| --- | --- | --- |
|  | *S&P 500* | *DJ Tech* |
| *QQQ* |  |  |
| *XLK* |  |  |
| *VGT* |  |  |
| *FDN* |  |  |
| *IYW* |  |  |

In conclusion, all of the coefficient is significant. But related to the R square, the regression based on the S&P 500 is higher than DJUSTC, which means the S&P can explain more the data than DJUSTC. The reason is the DJUSTC index just cover 100 hungry the company. Therefore, this index is not good enough as the market index.

For the multiple linear regression, we can get:

|  |  |
| --- | --- |
|  | Fama-French 3 factors |
| *QQQ* |  |
| *XLK* |  |
| *VGT* |  |
| *FDN* |  |
| *IYW* |  |

By regression, we discover in the Fama-French 3 factors, the SMB can be removed from the model. As the coefficient of the SMB is not significant. In this way, even remove it, it do not influence the accuracy of the model.

In addition, the that we obtain from the regression can be used to determine the performance of the investors. First, we can use get the Jemsen’s alpha from the regression. The formula of Jemsen’s alpha is :

R(i) - (R(f) + x (R(m) - R(f)))

We can see this function will equal to the obtain from the regression. But when want compare with others, just suable for the companies with the same . Other performance estimator is Treynor ratio, the formula is :

​​

This method is useful for the well-diversified companies to measure the performance. Because the represent the systemic risk, which cannot be eliminated by diversification. In other study. The ETF is suitable to adopt the Treynor ratio as the performance measurement.

**Reference**

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